# SL Paper 2

Tomek is attending a conference in Singapore. He has both trousers and shorts to wear. He also has the choice of wearing a tie or not.

The probability Tomek wears trousers is 0.3. If he wears trousers, the probability that he wears a tie is 0.8.
If Tomek wears shorts, the probability that he wears a tie is 0.15.
The following tree diagram shows the probabilities for Tomek's clothing options at the conference.
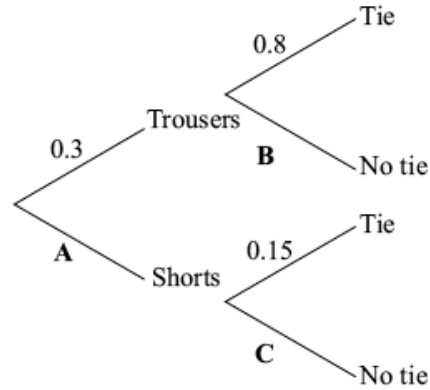
a. Find the value of   [3]

   (i)   A;

   (ii) B;

   (iii) C.

b. Calculate the probability that Tomek wears   [8]

   (i)   shorts and no tie;

   (ii) no tie;

   (iii) shorts given that he is not wearing a tie.

c. The conference lasts for two days.   [2]

   Calculate the probability that Tomek wears trousers on both days.

d. The conference lasts for two days.   [3]

   Calculate the probability that Tomek wears trousers on one of the days, and shorts on the other day.

A company performs an experiment on the efficiency of a liquid that is used to detect a nut allergy.

A group of 60 people took part in the experiment. In this group 26 are allergic to nuts. One person from the group is chosen at random.

A second person is chosen from the group.

When the liquid is added to a person's blood sample, it is expected to turn blue if the person is allergic to nuts and to turn red if the person is not allergic to nuts.

The company claims that the probability that the test result is correct is 98% for people who are allergic to nuts and 95% for people who are not allergic to nuts.

It is known that 6 in every 1000 adults are allergic to nuts.

This information can be represented in a tree diagram.
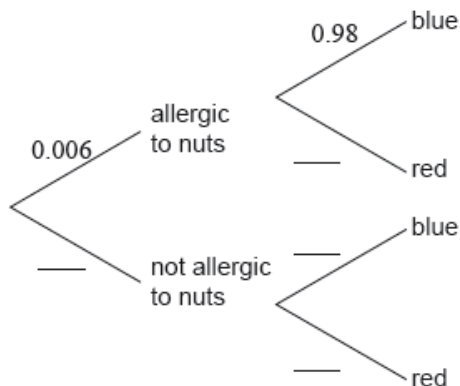
An adult, who was not part of the original group of 60, is chosen at random and tested using this liquid.

The liquid is used in an office to identify employees who might be allergic to nuts. The liquid turned blue for **38 employees**.

a. Find the probability that this person is **not** allergic to nuts. [2]

b. Find the probability that both people chosen are **not** allergic to nuts. [2]

c. **Copy** and complete the tree diagram. [3]

d. Find the probability that this adult is allergic to nuts and the liquid turns blue. [2]

e. Find the probability that the liquid turns blue. [3]

f. Find the probability that the tested adult is allergic to nuts given that the liquid turned blue. [3]

g. Estimate the number of employees, from this 38, who are allergic to nuts. [2]

Beartown has three local newspapers: *The Art Journal, The Beartown News,* and *The Currier.*

A survey shows that

32 % of the town's population read *The Art Journal,*

46 % read *The Beartown News,*

54 % read *The Currier,*

3 % read *The Art Journal* and *The Beartown News* **only,**

8 % read *The Art Journal* and *The Currier* **only,**

12 % read *The Beartown News* and *The Currier* **only,** and

5 % of the population reads **all** three newspapers.

a. Draw a Venn diagram to represent this information. Label *A* the set that represents *The Art Journal* readers, *B* the set that represents *The Beartown News* readers, and *C* the set that represents *The Currier* readers. [4]

b. Find the percentage of the population that does not read any of the three newspapers. [2]

c. Find the percentage of the population that reads exactly one newspaper. [2]

d. Find the percentage of the population that reads *The Art Journal* or *The Beartown News* but not *The Currier*. [2]

e. A local radio station states that 83 % of the population reads either *The Beartown News* or *The Currier*. [2]

    Use your Venn diagram to decide whether the statement is true. Justify your answer.

f. The population of Beartown is 120 000. The local radio station claimed that 34 000 of the town's citizens read at least two of the local [4]

    newspapers.

    Find the percentage error in this claim.

A survey of 100 families was carried out, asking about the pets they own. The results are given below.

   56 owned dogs *(S)*
   38 owned cats *(Q)*
   22 owned birds *(R)*
   16 owned dogs and cats, but not birds
   8 owned birds and cats, but not dogs
   3 owned dogs and birds, but not cats
   4 owned all three types of pets

a. Draw a Venn diagram to represent this information. [5]

b. Find the number of families who own no pets. [2]

c. Find the percentage of families that own exactly one pet. [3]

d. A family is chosen at random. Find the probability that they own a cat, given that they own a bird. [2]

Pam has collected data from a group of 400 IB Diploma students about the Mathematics course they studied and the language in which they were examined (English, Spanish or French). The summary of her data is given below.

	Mathematics HL	Mathematics SL	Mathematical Studies SL	Total
English	50	70	80	200
Spanish	30	50	30	110
French	20	30	40	90
Total	100	150	150	400

a. A student is chosen at random from the group. Find the probability that the student [8]

   (i)   studied Mathematics HL;

   (ii)   was examined in French;

   (iii) studied Mathematics HL and was examined in French;

   (iv) did not study Mathematics SL and was not examined in English;

   (v)   studied Mathematical Studies SL given that the student was examined in Spanish.

b. Pam believes that the Mathematics course a student chooses is independent of the language in which the student is examined. [2]

Using your answers to parts (a) (i), (ii) and (iii) above, state whether there is any evidence for Pam's belief. Give a reason for your answer.

c. Pam decides to test her belief using a Chi-squared test at the 5% level of significance. [3]

   (i)   State the null hypothesis for this test.

   (ii)   Show that the expected number of Mathematical Studies SL students who took the examination in Spanish is 41.3, correct to 3 significant figures.

d. Write down [4]

   (i)   the Chi-squared calculated value;

   (ii)   the number of degrees of freedom;

   (iii) the Chi-squared critical value.

e. State, giving a reason, whether there is sufficient evidence at the 5% level of significance that Pam's belief is correct. [2]

Leanne goes fishing at her favourite pond. The pond contains four different types of fish: bream, flathead, whiting and salmon. The fish are either undersized or normal. This information is shown in the table below.

Size / Type of fish	Bream	Flathead	Whiting	Salmon	Total
Undersized	3	12	18	9	42
Normal	0	11	24	13	48
Total	3	23	42	22	

a. Write down the total number of fish in the pond. [1]

b. Leanne catches a fish. [7]
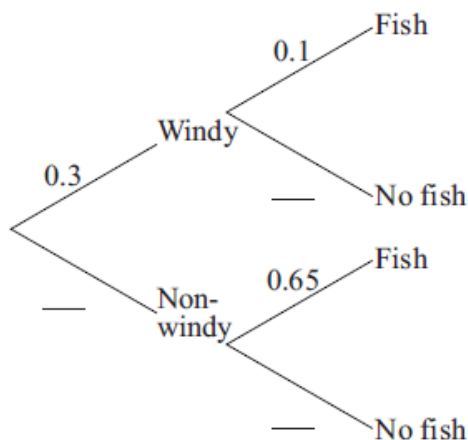
Find the probability that she

(i) catches an undersized bream;

(ii) catches either a flathead or an undersized fish or both;

(iii) does not catch an undersized whiting;

(iv) catches a whiting given that the fish was normal.

c. Leanne notices that on windy days, the probability she catches a fish is 0.1 while on non-windy days the probability she catches a fish is 0.65. [3]

The probability that it will be windy on a particular day is 0.3.

Copy and complete the probability tree diagram below.

d. Leanne notices that on windy days, the probability she catches a fish is 0.1 while on non-windy days the probability she catches a fish is 0.65. [2]

The probability that it will be windy on a particular day is 0.3.

Calculate the probability that it is windy and Leanne catches a fish on a particular day.

e. Leanne notices that on windy days, the probability she catches a fish is 0.1 while on non-windy days the probability she catches a fish is 0.65. [3]

The probability that it will be windy on a particular day is 0.3.

Calculate the probability that Leanne catches a fish on a particular day.

f. Use your answer to part (e) to calculate the probability that Leanne catches a fish on two consecutive days. [2]

g. Leanne notices that on windy days, the probability she catches a fish is 0.1 while on non-windy days the probability she catches a fish is 0.65. [3]

The probability that it will be windy on a particular day is 0.3.

Given that Leanne catches a fish on a particular day, calculate the probability that the day was windy.

A geometric sequence has second term 12 and fifth term 324.

Consider the following propositions

p: The number is a multiple of five.

q: The number is even.

r: The number ends in zero.

i, a.Calculate the value of the common ratio. [4]

i, b.Calculate the 10th term of this sequence. [3]

i, c.The kth term is the first term which is greater than 2000. Find the value of k. [3]

ii, a.Write in words $(q \wedge \neg r) \Rightarrow \neg p$. [3]

ii, b.Consider the statement "If the number is a multiple of five, and is not even then it will not end in zero". [4]

Write this statement in symbolic form.

ii, b.Consider the statement "If the number is a multiple of five, and is not even then it will not end in zero". [2]

Write the contrapositive of this statement in symbolic form.

In a company it is found that 25 % of the employees encountered traffic on their way to work. From those who encountered traffic the probability of

being late for work is 80 %.

From those who did not encounter traffic, the probability of being late for work is 15 %.

The tree diagram illustrates the information.
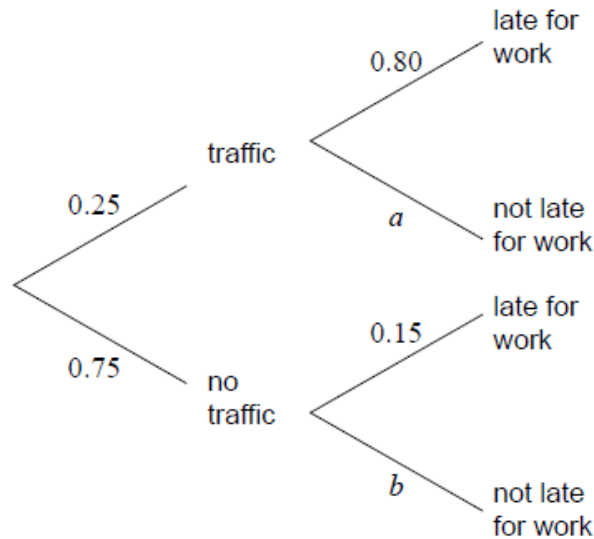
The company investigates the different means of transport used by their employees in the past year to travel to work. It was found that the three most

common means of transport used to travel to work were public transportation (P), car (C) and bicycle (B).

The company finds that 20 employees travelled by car, 28 travelled by bicycle and 19 travelled by public transportation in the last year.

Some of the information is shown in the Venn diagram.
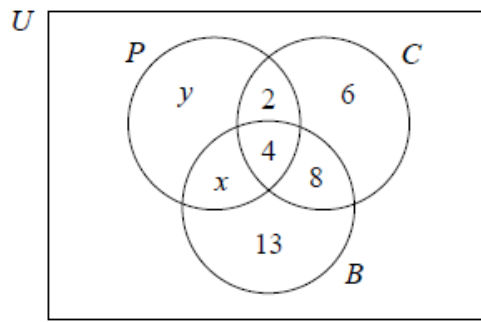
There are 54 employees in the company.

a.i. Write down the value of a. [1]

a.ii. Write down the value of b. [1]

b.i. Use the tree diagram to find the probability that an employee encountered traffic and was late for work. [2]

b.ii. Use the tree diagram to find the probability that an employee was late for work. [3]

b.iii. Use the tree diagram to find the probability that an employee encountered traffic given that they were late for work. [3]

c.i. Find the value of x. [1]

c.ii. Find the value of y. [1]

d. Find the number of employees who, in the last year, did not travel to work by car, bicycle or public transportation. [2]

e. Find $n\left((C \cup B) \cap P'\right)$. [2]

Consider these three propositions, in which x is a natural number.

$$p: x \text{ is a factor of } 60$$
$$q: x \text{ is a multiple of } 4$$
$$r: x \text{ is a multiple of } 5$$

a. Write down in symbolic form the compound proposition [3]

"If x is a factor of 60 then x is a multiple of 5 or x is not a multiple of 4."

b. Write down in words the compound proposition $\neg r \wedge (p \veebar q)$. [3]

c. **Copy** the following truth table and complete the last three columns. [3]

p	q	r	$\neg r$	$p \veebar q$	$\neg r \wedge (p \veebar q)$
T	T	T			
T	T	F			
T	F	T			
T	F	F			T
F	T	T			
F	T	F			
F	F	T			
F	F	F			

d. State why the compound proposition $\neg r \wedge (p \veebar q)$ is not a logical contradiction. [1]

e. A row from the truth table from part (c) is given below. [1]

p	q	r	$\neg r$	$p \veebar q$	$\neg r \wedge (p \veebar q)$
T	F	F			T

Write down **one** value of x that satisfies these truth values.

Forty families were surveyed about the places they went to on the weekend. The places were the circus (C), the museum (M) and the park (P).

16 families went to the circus

22 families went to the museum

14 families went to the park

4 families went to all three places

7 families went to both the circus and the museum, but not the park

3 families went to both the circus and the park, but not the museum

1 family went to the park only

a. Draw a Venn diagram to represent the given information using sets labelled C, M and P. Complete the diagram to include the number of families [4] represented in each region.

b. Find the number of families who [4]

(i) went to the circus only;

(ii) went to the museum and the park but not the circus;

(iii) did not go to any of the three places on the weekend.

c. A family is chosen at random from the group of 40 families. Find the probability that the family went to [8]

(i) the circus;

(ii) two or more places;

(iii) the park or the circus, but not the museum;

    (iv) the museum, given that they also went to the circus.

d. Two families are chosen at random from the group of 40 families.   [3]

    Find the probability that both families went to the circus.

A group of tourists went on safari to a game reserve. The game warden wanted to know how many of the tourists saw Leopard (L), Cheetah (C)

or Rhino (R). The results are given as follows.

    5 of the tourists saw all three
    7 saw Leopard and Rhino
    1 saw Cheetah and Leopard **but not** Rhino
    4 saw Leopard **only**
    3 saw Cheetah **only**
    9 saw Rhino **only**

a. Draw a Venn diagram to show this information.   [4]

b. There were 25 tourists in the group and every tourist saw at least one of the three types of animal.   [2]

    Find the number of tourists that saw Cheetah and Rhino **but not** Leopard.

c. There were 25 tourists in the group and every tourist saw at least one of the three types of animal.   [6]

    Calculate the probability that a tourist chosen at random from the group

    (i)   saw Leopard;

    (ii)   saw **only one** of the three types of animal;

    (iii)   saw **only** Leopard, given that he saw only one of the three types of animal.

d. There were 25 tourists in the group and every tourist saw at least one of the three types of animal.   [2]

    If a tourist chosen at random from the group saw Leopard, find the probability that he also saw Cheetah.

Consider the following statements.

    p: the land has been purchased
    q: the building permit has been obtained
    r: the land can be used for residential purposes

a. Write the following argument in symbolic form.   [3]

    "If the land has been purchased and the building permit has been obtained, then the land can be used for residential purposes."

b. **In your answer booklet**, copy and complete a truth table for the argument in part (a).   [2]

    Begin your truth table as follows.

p	q	r	
T	T	T	
T	T	F	
T	F	T	
T	F	F	
F	T	T	
F	T	F	
F	F	T	
F	F	F	

c. Use your truth table to determine whether the argument in part (a) is valid. [2]

   Give a reason for your decision.

d. Write down the inverse of the argument in part (a) [4]

   (i)   in symbolic form;

   (ii) in words.

A group of 120 women in the USA were asked whether they had visited the continents of Europe (E) or South America (S) or Asia (A).

   7 had visited all three continents

   28 had visited Europe only

   22 had visited South America only

   16 had visited Asia only

   15 had visited Europe and South America but had not visited Asia

   x had visited South America and Asia but had not visited Europe

   $2x$ had visited Europe and Asia but had not visited South America

   20 had not visited any of these continents

a. Draw a Venn diagram, using sets labelled E, S and A, to show this information. [5]

b. Calculate the value of x. [2]

c. Explain, in words, the meaning of $(E \cup S) \cap A'$. [2]

d. Write down $n\left((E \cup S \cup A)'\right)$. [1]

e. Find the probability that a woman selected at random from the group had visited Europe. [2]

f. Find the probability that a woman selected at random from the group had visited Europe, given that she had visited Asia. [2]

g. Two women from the group are selected at random. [3]

   Find the probability that both women selected had visited South America.

A group of 66 people went on holiday to Hawaii. During their stay, three trips were arranged: a boat trip (B), a coach trip (C) and a helicopter trip (H).

From this group of people:

> 3 went on all three trips;
> 16 went on the coach trip **only**;
> 13 went on the boat trip **only**;
> 5 went on the helicopter trip **only**;
> x went on the coach trip and the helicopter trip **but not** the boat trip;
> $2x$ went on the boat trip and the helicopter trip **but not** the coach trip;
> $4x$ went on the boat trip and the coach trip **but not** the helicopter trip;
> 8 did not go on any of the trips.

One person in the group is selected at random.

a. Draw a Venn diagram to represent the given information, using sets labelled B, C and H. [5]

b. Show that $x = 3$. [2]

c. Write down the value of $n(B \cap C)$. [1]

d. Find the probability that this person [4]

   (i)   went on at most one trip;

   (ii)   went on the coach trip, given that this person also went on both the helicopter trip and the boat trip.


Mike, the laboratory mouse, is placed at the starting point, S, of a maze. Some paths in the maze lead to Trap A, some to Trap B, and others to escape doors. Some paths have one and some have two sections. If his path forks, Mike randomly chooses a path **forward**.

The following tree diagram represents the maze, showing all possible paths, and the probability that Mike chooses a certain section of a path through the maze.
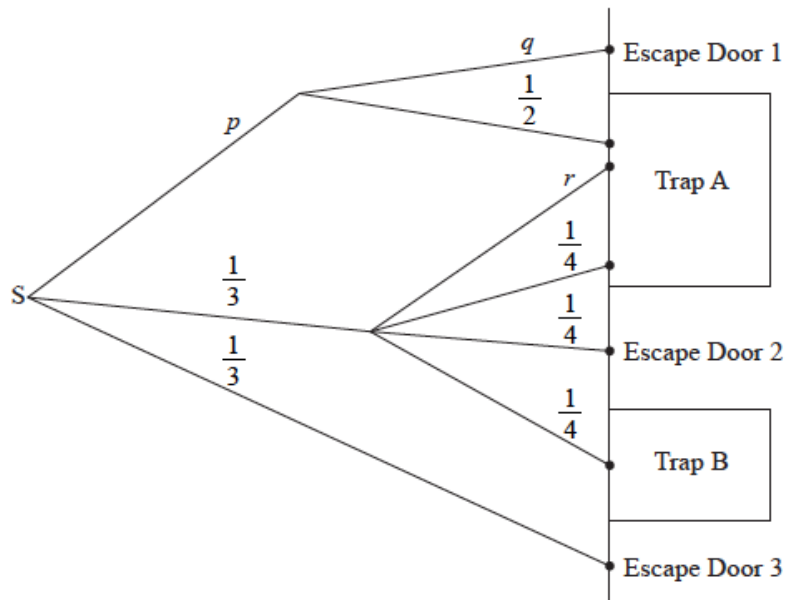
a. Write down the value of [3]

    (i) p;

    (ii) q;

    (iii) r.

b. (i)   Find the probability that Mike reaches Trap B. [7]

    (ii)   Find the probability that Mike reaches Trap A.

    (iii)   Find the probability that Mike escapes from the maze.

c. Sonya, a lab assistant, counts the number of paths that lead to traps or escape doors. She believes that the probability that Mike will be [2]

   trapped is greater than the probability that he will escape.

   State whether Sonya is correct. Give a mathematical justification for your conclusion.

d. During the first trial Mike escapes. [3]

   Given that Mike escaped, find the probability that he went directly from S to Escape Door 3.

A water container is made in the shape of a cylinder with internal height h cm and internal base radius r cm.
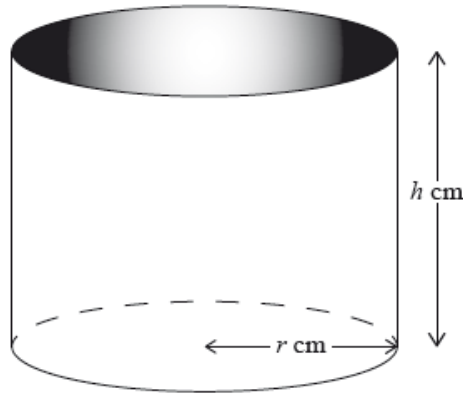
The water container has no top. The inner surfaces of the container are to be coated with a water-resistant material.

The volume of the water container is 0.5 m^3.

The water container is designed so that the area to be coated is minimized.

One can of water-resistant material coats a surface area of 2000 cm^2.

a. Write down a formula for A, the surface area to be coated. [2]

b. Express this volume in cm^3. [1]

c. Write down, in terms of r and h, an equation for the volume of this water container. [1]

d. Show that $A = \pi r^2 \frac{1\,000\,000}{r}$. [2]

d. Show that $A = \pi r^2 + \frac{1\,000\,000}{r}$. [2]

e. Find $\frac{\mathrm{d}A}{\mathrm{d}r}$. [3]

f. Using your answer to part (e), find the value of r which minimizes A. [3]

g. Find the value of this minimum area. [2]

h. Find the least number of cans of water-resistant material that will coat the area in part (g). [3]

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50 students at Rambling High School were asked how they travelled to school yesterday. All of the students travelled by bus, by car or walked.

12 students travelled by car only
7 students travelled by bus only
5 students travelled by car and walked, but did not use a bus
10 students travelled by bus and walked, but did not use a car
3 students used all three forms of travel.

a. Represent this information on a Venn Diagram. [4]

b. There were 28 students who used a bus to travel to school. Calculate the number of students [4]

(i)   who travelled by car and by bus but did not walk;

(ii) who travelled by car.

c. Tomoko used a bus to travel to school yesterday. [2]

Find the probability that she also walked.

d. Two students are chosen at random from all 50 students. [7]

Find the probability that
(i)   both students walked;
(ii) only one of the students walked.

When Geraldine travels to work she can travel either by car (C), bus (B) or train (T). She travels by car on one day in five. She uses the bus 50 %

of the time. The probabilities of her being late (L) when travelling by car, bus or train are 0.05, 0.12 and 0.08 respectively.

*It is **not** necessary to use graph paper for this question.*

i.a. Copy the tree diagram below and fill in all the probabilities, where *NL* represents not late, to represent this information. [5]
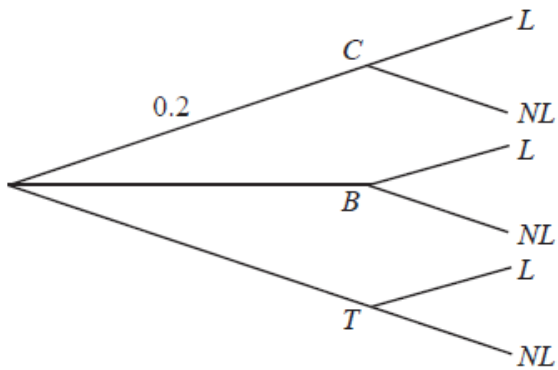
i.b. Find the probability that Geraldine travels by bus and is late. [1]

i.c. Find the probability that Geraldine is late. [3]

i.d. Find the probability that Geraldine travelled by train, given that she is late. [3]

ii.a. Sketch the curve of the function $f(x) = x^3 - 2x^2 + x - 3$ for values of x from –2 to 4, giving the intercepts with both axes. [3]

ii.b. On the same diagram, sketch the line $y = 7 - 2x$ and find the coordinates of the point of intersection of the line with the curve. [3]

ii.c. Find the value of the gradient of the curve where $x = 1.7$. [2]

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100 students at IB College were asked whether they study Music (M), Chemistry (C), or Economics (E) with the following results.

10 study all three

15 study Music and Chemistry

17 study Music and Economics

12 study Chemistry and Economics

11 study Music **only**

6 study Chemistry **only**

a. Draw a Venn diagram to represent the information above. [4]

b. Write down the number of students who study Music but not Economics. [1]

c. There are 22 Economics students **in total**. [4]

(i) Calculate the number of students who study Economics only.

(ii) Find the number of students who study none of these three subjects.

d. A student is chosen at random from the 100 that were asked above. [7]

Find the probability that this student

(i) studies Economics;

(ii) studies Music and Chemistry but not Economics;

(iii) does not study either Music or Economics;

(iv) does not study Music given that the student does not study Economics.

Jorge conducted a survey of 200 drivers. He asked two questions:

How long have you had your driving licence?
Do you wear a seat belt when driving?

The replies are summarized in the table below.

	Wear a seat belt	Do not wear a seat belt
Licence less than 2 years	38	42
Licence between 2 and 15 years	30	45
Licence more than 15 years	30	15

a. Jorge applies a χ^2 test at the 5% level to investigate whether wearing a seat belt is associated with the time a driver has had their licence.   [8]

(i)   Write down the null hypothesis, H_0.

(ii)   Write down the number of degrees of freedom.

(iii)   Show that the expected number of drivers that wear a seat belt and have had their driving licence for more than 15 years is 22, correct to the nearest whole number.

(iv)   Write down the χ^2 test statistic for this data.

(v)   Does Jorge accept H_0 ? Give a reason for your answer.

b. Consider the 200 drivers surveyed. One driver is chosen at random. Calculate the probability that   [4]

(i)   this driver wears a seat belt;

(ii)   the driver does not wear a seat belt, **given that** the driver has held a licence for more than 15 years.

c. Two drivers are chosen at random. Calculate the probability that   [6]

(i)   both wear a seat belt.

(ii)   at least one wears a seat belt.

In a college 450 students were surveyed with the following results

150 have a television

205 have a computer

220 have an iPhone

75 have an iPhone and a computer

60 have a television and a computer

*70 have a television and an iPhone*

40 have all three.

a. Draw a Venn diagram to show this information. Use *T* to represent the set of students who have a television, *C* the set of students who have a computer and *I* the set of students who have an iPhone. [4]

b. Write down the number of students that [2]

   (i) have a computer only;

   (ii) have an iPhone and a computer but no television.

c. Write down $n[T \cap (C \cup I)']$. [1]

d. Calculate the number of students who have none of the three. [2]

e. Two students are chosen at random from the 450 students. Calculate the probability that [6]

   (i) neither student has an iPhone;

   (ii) only one of the students has an iPhone.

f. The students are asked to collect money for charity. In the first month, the students collect *x* dollars and the students collect *y* dollars in each [3]

   subsequent month. In the first 6 months, they collect 7650 dollars. This can be represented by the equation $x + 5y = 7650$.

   In the first 10 months they collect 13 050 dollars.

   (i) Write down a second equation in *x* and *y* to represent this information.

   (ii) Write down the value of *x* and of *y* .

g. The students are asked to collect money for charity. In the first month, the students collect *x* dollars and the students collect *y* dollars in each [3]

   subsequent month. In the first 6 months, they collect 7650 dollars. This can be represented by the equation $x + 5y = 7650$.

   In the first 10 months they collect 13 050 dollars.

   Calculate the number of months that it will take the students to collect 49 500 dollars.

The figure below shows the lengths in centimetres of fish found in the net of a small trawler.
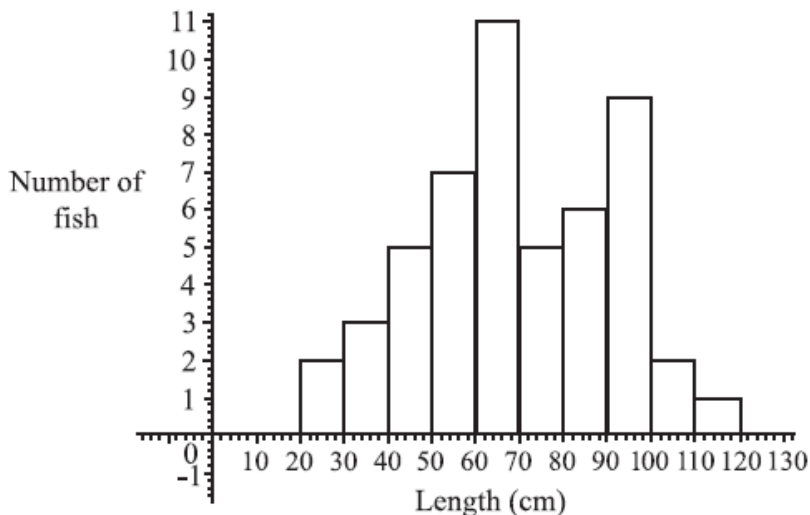
a. Find the total number of fish in the net. [2]

b. Find (i) the modal length interval,   [5]

   (ii) the interval containing the median length,

   (iii) an estimate of the mean length.

c. (i) Write down an estimate for the standard deviation of the lengths.   [3]

   (ii) How many fish (if any) have length **greater than** three standard deviations **above** the mean?

d. The fishing company must pay a fine if more than 10% of the catch have lengths less than 40cm.   [2]

   Do a calculation to decide whether the company is fined.

e. A sample of 15 of the fish was weighed. The weight, W was plotted against length, L as shown below. [2]
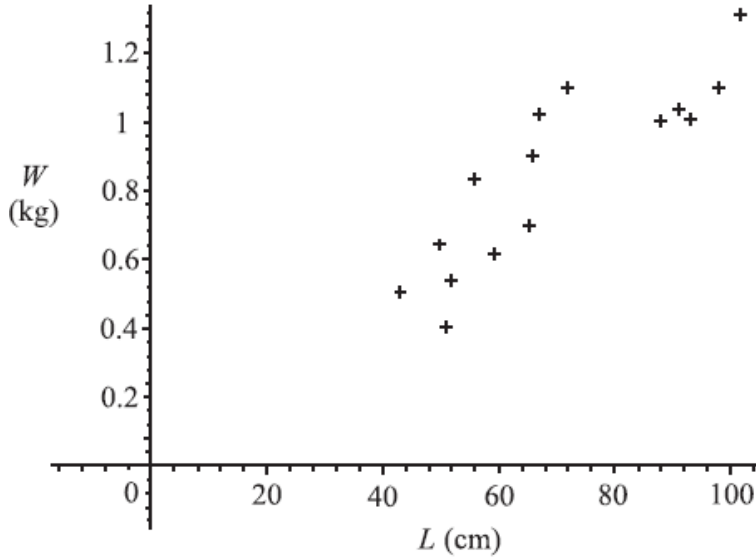

Exactly **two** of the following statements about the plot could be correct. Identify the two correct statements.

Note: You do **not** need to enter data in a GDC **or** to calculate r exactly.

(i) The value of r, the correlation coefficient, is approximately 0.871.

(ii) There is an exact linear relation between W and L.

(iii) The line of regression of W on L has equation $W = 0.012L + 0.008$.

(iv) There is negative correlation between the length and weight.

(v) The value of r, the correlation coefficient, is approximately 0.998.

(vi) The line of regression of W on L has equation $W = 63.5L + 16.5$.

Phoebe chooses a biscuit from a blue tin on a shelf. The tin contains one chocolate biscuit and four plain biscuits. She eats the biscuit and chooses another one from the tin. The tree diagram below represents the situation with the four possible outcomes where A stands for chocolate biscuit and B for plain biscuit.
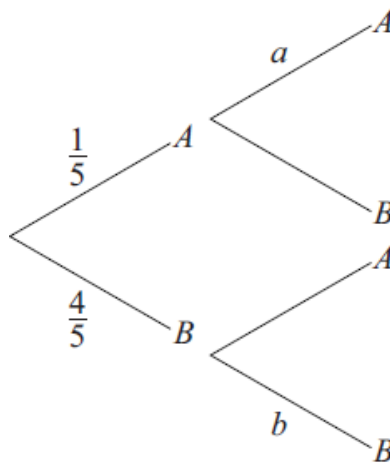
On another shelf there are two tins, one red and one green. The red tin contains three chocolate biscuits and seven plain biscuits and the green tin contains one chocolate biscuit and four plain biscuits. Andrew randomly chooses either the red or the green tin and randomly selects a biscuit.

a.i. Write down the value of *a*. [1]

a.ii. Write down the value of *b*. [2]

a.iii Find the probability that both biscuits are plain. [3]

b. **Copy and complete** the tree diagram below. [3]
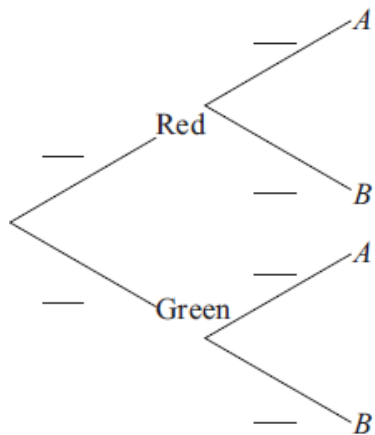
c.i. Find the probability that he chooses a chocolate biscuit. [3]

c.ii. Find the probability that he chooses a biscuit from the red tin given that it is a chocolate biscuit. [3]

A store recorded their sales of televisions during the 2010 football World Cup. They looked at the numbers of televisions bought by gender and the size of the television screens.

This information is shown in the table below; S represents the size of the television screen in inches.

	$S \leq 22$	$22 < S \leq 32$	$32 < S \leq 46$	$S > 46$	Total
Female	65	100	40	15	220
Male	20	65	140	55	280
Total	85	165	180	70	500

The store wants to use this information to predict the probability of selling these sizes of televisions for the 2014 football World Cup.

a. Use the table to find the probability that [6]

(i) a television will be bought by a female;

(ii) a television with a screen size of $32 < S \leq 46$ will be bought;

(iii) a television with a screen size of $32 < S \leq 46$ will be bought by a female;

(iv) a television with a screen size greater than 46 inches will be bought, given that it is bought by a male.

b. The manager of the store wants to determine whether the screen size is independent of gender. A Chi-squared test is performed at the 1 % [1] significance level.

Write down the null hypothesis.

c. The manager of the store wants to determine whether the screen size is independent of gender. A Chi-squared test is performed at the 1 % [2] significance level.

Show that the expected frequency for females who bought a screen size of $32 < S \leq 46$, is 79, correct to the nearest integer.

d. The manager of the store wants to determine whether the screen size is independent of gender. A Chi-squared test is performed at the 1 % [1] significance level.

Write down the number of degrees of freedom.

e. The manager of the store wants to determine whether the screen size is independent of gender. A Chi-squared test is performed at the 1 % [2] significance level.

Write down the χ^2 calculated value.

f. The manager of the store wants to determine whether the screen size is independent of gender. A Chi-squared test is performed at the 1 % [1] significance level.

Write down the critical value for this test.

g. The manager of the store wants to determine whether the screen size is independent of gender. A Chi-squared test is performed at the 1 % [2] significance level.

Determine if the null hypothesis should be accepted. Give a reason for your answer.

A group of 50 students completed a questionnaire for a Mathematical Studies project. The following data was collected.

18 students own a digital camera (D)
15 students own an iPod (I)
26 students own a cell phone (C)
1 student owns all three items
5 students own a digital camera and an iPod but not a cell phone
2 students own a cell phone and an iPod but not a digital camera
3 students own a cell phone and a digital camera but not an iPod

Claire and Kate both wish to go to the cinema but one of them has to stay at home to baby-sit.

The probability that Kate goes to the cinema is 0.2. If Kate does not go Claire goes.

If Kate goes to the cinema the probability that she is late home is 0.3.

If Claire goes to the cinema the probability that she is late home is 0.6.

i.a. Represent this information on a Venn diagram.   [4]

i.b. Calculate the number of students who own none of the items mentioned above.   [2]

i.c. If a student is chosen at random, write down the probability that the student owns a digital camera **only**.   [1]

i.d. If two students are chosen at random, calculate the probability that they both own a cell phone **only**.   [3]

i.e. If a student owns an iPod, write down the probability that the student also owns a digital camera.   [2]

ii.a. Copy and complete the probability tree diagram below.   [3]
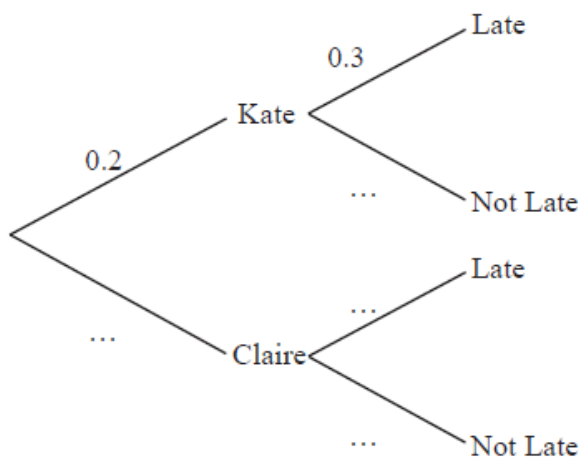
ii.b. Calculate the probability that   [5]

(i)   Kate goes to the cinema and is not late;

(ii) the person who goes to the cinema arrives home late.

One day the numbers of customers at three cafés, "Alan's Diner" (A), "Sarah's Snackbar" (S) and "Pete's Eats" (P), were recorded and are given below.

17 were customers of Pete's Eats only
27 were customers of Sarah's Snackbar only

15 were customers of Alan's Diner only
10 were customers of Pete's Eats **and** Sarah's Snackbar **but not** Alan's Diner
8 were customers of Pete's Eats **and** Alan's Diner **but not** Sarah's Snackbar

Some of the customers in each café were given survey forms to complete to find out if they were satisfied with the standard of service they received.

	Pete's Eats	Alan's Diner	Sarah's Snackbar	Total
Dissatisfied	16	8	16	40
Satisfied	26	20	34	80
Total	42	28	50	120

A.a Draw a Venn Diagram, using sets labelled A, S and P, that shows this information. [3]

A.b There were 48 customers of Pete's Eats that day. Calculate the number of people who were customers of all three cafés. [2]

A.c There were 50 customers of Sarah's Snackbar that day. Calculate the total number of people who were customers of Alan's Diner. [3]

A.d Write down the number of customers of Alan's Diner that were also customers of Pete's Eats. [1]

A.e Find $n[(S \cup P) \cap A']$. [2]

B.a One of the survey forms was chosen at random, find the probability that the form showed "Dissatisfied"; [2]

B.b One of the survey forms was chosen at random, find the probability that the form showed "Satisfied" and was completed at Sarah's Snackbar; [2]

B.c One of the survey forms was chosen at random, find the probability that the form showed "Dissatisfied", given that it was completed at Alan's [2]

   Diner.

B.d A χ^2 test at the 5% significance level was carried out to determine whether there was any difference in the level of customer satisfaction in each [1]

   of the cafés.

   Write down the null hypothesis, H_0, for the χ^2 test.

B.e A χ^2 test at the 5% significance level was carried out to determine whether there was any difference in the level of customer satisfaction in each [1]

   of the cafés.

   Write down the number of degrees of freedom for the test.

B.f A χ^2 test at the 5% significance level was carried out to determine whether there was any difference in the level of customer satisfaction in each [2]

   of the cafés.

   Using your graphic display calculator, find χ^2_{calc}.

B.g A χ^2 test at the 5% significance level was carried out to determine whether there was any difference in the level of customer satisfaction in each [2]

   of the cafés.

   State, giving a reason, the conclusion to the test.

The Venn diagram below represents the students studying Mathematics (A), Further Mathematics (B) and Physics (C) in a school.

    50 students study Mathematics

    38 study Physics

    20 study Mathematics and Physics but not Further Mathematics

    10 study Further Mathematics but not Physics

    12 study Further Mathematics and Physics

    6 study Physics but not Mathematics

    3 study none of these three subjects.
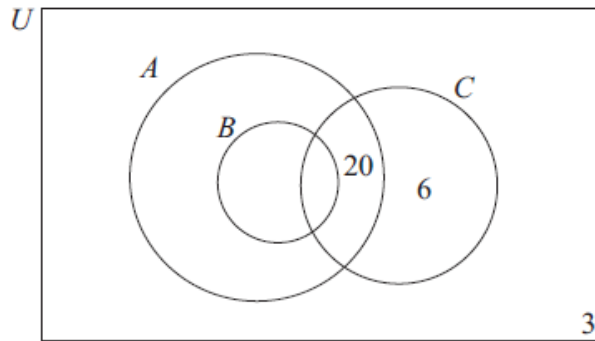
Three propositions are given as

$$p : \text{It is snowing} \quad q : \text{The roads are open} \quad r : \text{We will go skiing}$$

A, a Copy and complete the Venn diagram **on your answer paper**.   [3]

A, b Write down the number of students who study Mathematics but not Further Mathematics.   [1]

A, c Write down the total number of students in the school.   [1]

A, d Write down $n(\mathrm{B} \cup \mathrm{C})$.   [2]

B, a Write the following compound statement in symbolic form.   [2]

    "It is snowing and the roads are not open."

B, b Write the following compound statement in words.   [3]

    $(\neg p \wedge q) \Rightarrow r$

B, c An incomplete truth table for the compound proposition $(\neg p \wedge q) \Rightarrow r$ is given below.   [3]

    Copy and complete the truth table **on your answer paper**.

p	q	r	$\neg p$	$\neg p \wedge q$	$(\neg p \wedge q) \Rightarrow r$
T	T	T			
T	T	F			
T	F	T			
T	F	F			
F	T	T			
F	T	F			
F	F	T			
F	F	F			

Part A

100 students are asked what they had for breakfast on a particular morning. There were three choices: cereal (X) , bread (Y) and fruit (Z). It is found that

10 students had all three

17 students had bread and fruit only

15 students had cereal and fruit only

12 students had cereal and bread only

13 students had only bread

8 students had only cereal

9 students had only fruit

Part B

The same 100 students are also asked how many meals on average they have per day. The data collected is organized in the following table.

	3 or fewer meals per day	4 or 5 meals per day	More than 5 meals per day	Total
Male	15	25	15	55
Female	12	20	13	45
Total	27	45	28	100

A χ^2 test is carried out at the 5 % level of significance.

A.aRepresent this information on a Venn diagram. [4]

A.bFind the number of students who had none of the three choices for breakfast. [2]

A.cWrite down the percentage of students who had fruit for breakfast. [2]

A.dDescribe in words what the students in the set $X \cap Y'$ had for breakfast. [2]

A.e Find the probability that a student had **at least** two of the three choices for breakfast. [2]

A.f. Two students are chosen at random. Find the probability that both students had all three choices for breakfast. [3]

B.a Write down the null hypothesis, H_0, for this test. [1]

B.b Write down the number of degrees of freedom for this test. [1]

B.c Write down the critical value for this test. [1]

B.d Show that the expected number of females that have more than 5 meals per day is 13, correct to the nearest integer. [2]

B.e Use your graphic display calculator to find the χ^2_{calc} for this data. [2]

B.f. Decide whether H_0 must be accepted. Justify your answer. [2]

Sharon and Lisa share a flat. Sharon cooks dinner three nights out of ten. If Sharon does not cook dinner, then Lisa does. If Sharon cooks dinner the probability that they have pasta is 0.75. If Lisa cooks dinner the probability that they have pasta is 0.12.

A survey was carried out in a year 12 class. The pupils were asked which pop groups they like out of the *Rockers* (*R*), the *Salseros* (*S*), and the *Bluers* (*B*). The results are shown in the following diagram.
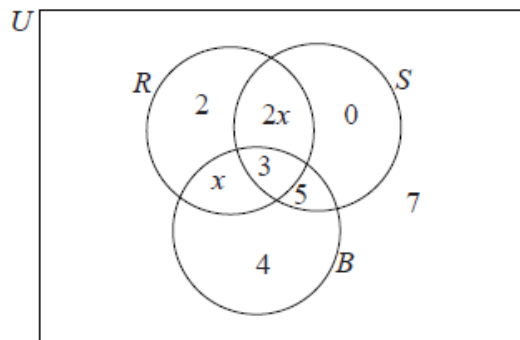
i, a **Copy and complete** the tree diagram to represent this information. [3]

Pasta

0.75

Sharon

_____

Not Pasta

Pasta

_____

Lisa

_____

Not Pasta
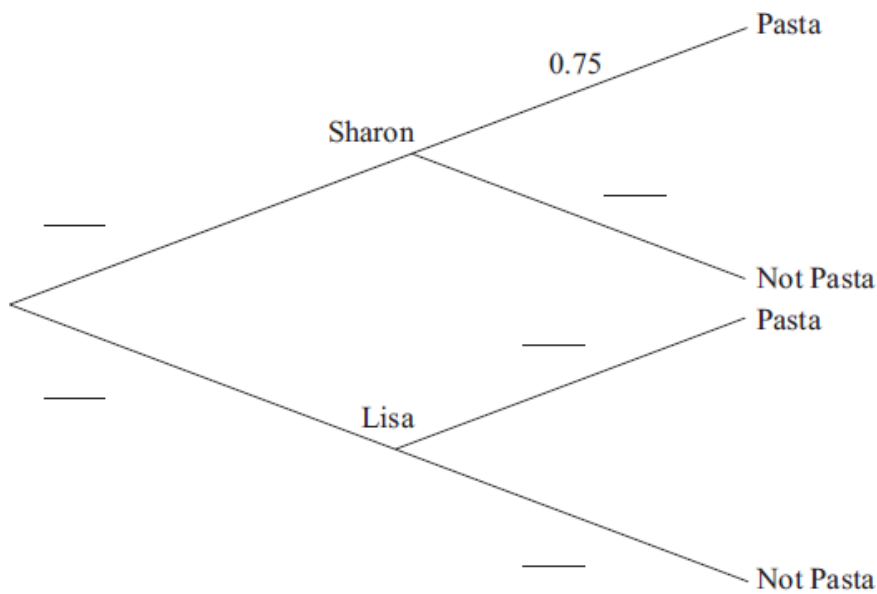
i, b Find the probability that Lisa cooks dinner and they do not have pasta. [2]

i, c Find the probability that they do not have pasta. [3]

i, d Given that they do not have pasta, find the probability that Lisa cooked dinner. [3]

ii, a Write down $n(R \cap S \cap B)$. [1]

ii, b Find $n(R')$. [2]

ii, c Describe which groups the pupils in the set $S \cap B$ like. [2]

ii, d Use set notation to describe the group of pupils who like the *Rockers* and the *Bluers* but do not like the *Salseros*. [2]

ii, e There are 33 pupils in the class. [2]

Find *x*.

ii, e There are 33 pupils in the class. [1]

Find the number of pupils who like the *Rockers*.

The seniors from Gulf High School are required to participate in exactly one after-school sport. Data were gathered from a sample of 120 students regarding their choice of sport. The following data were recorded.

Gender	Sport			
	Football	Tennis	Basketball	Total
Male	17	8	10	35
Female	31	17	37	85
Total	48	25	47	120

A χ^2 test was carried out at the 5 % significance level to analyse the relationship between gender and choice of after-school sport.

a. Write down the null hypothesis, H_0, for this test. [1]

b. Find the expected value of female footballers. [2]

c. Write down the number of degrees of freedom. [1]

d. Write down the critical value of χ^2, at the 5 % level of significance. [1]

e. Use your graphic display calculator to determine the χ^2_{calc} value. [2]

f. Determine whether H_0 should be accepted. Justify your answer. [2]

g. One student is chosen at random from the 120 students. [2]

Find the probability that this student

(i) is male;

(ii) plays tennis.

h. Two students are chosen at random from the 120 students. [5]

Find the probability that

(i) both play football;

(ii) neither play basketball.

A group of 100 customers in a restaurant are asked which fruits they like from a choice of mangoes, bananas and kiwi fruits. The results are as

follows.

15 like all three fruits

22 like mangoes and bananas

33 like mangoes and kiwi fruits

27 like bananas and kiwi fruits

8 like none of these three fruits

x like **only** mangoes

a. **Copy** the following Venn diagram and correctly insert all values from the above information. [3]
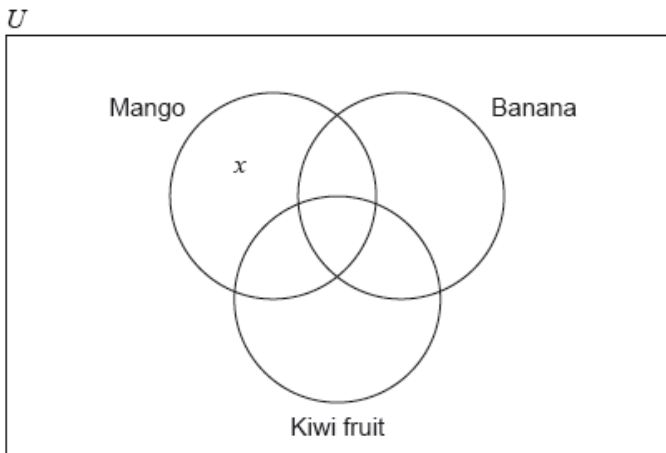
b. The number of customers that like **only** mangoes is equal to the number of customers that like **only** kiwi fruits. This number is half of the [2]

  number of customers that like **only** bananas.

  Complete your Venn diagram from part (a) with this additional information **in terms of** x.

c. The number of customers that like **only** mangoes is equal to the number of customers that like **only** kiwi fruits. This number is half of the [2]

  number of customers that like **only** bananas.

  Find the value of x.

d. The number of customers that like **only** mangoes is equal to the number of customers that like **only** kiwi fruits. This number is half of the [2]

  number of customers that like **only** bananas.

  Write down the number of customers who like

  (i)   mangoes;

  (ii)   mangoes or bananas.

e. The number of customers that like **only** mangoes is equal to the number of customers that like **only** kiwi fruits. This number is half of the [4]

  number of customers that like **only** bananas.

 A customer is chosen at random from the 100 customers. Find the probability that this customer

  (i)   likes none of the three fruits;

  (ii)   likes only two of the fruits;

  (iii)   likes all three fruits given that the customer likes mangoes and bananas.

f. The number of customers that like **only** mangoes is equal to the number of customers that like **only** kiwi fruits. This number is half of the [3]

  number of customers that like **only** bananas.

  Two customers are chosen at random from the 100 customers. Find the probability that the two customers like none of the three fruits.

Neil has three dogs. Two are brown and one is grey. When he feeds the dogs, Neil uses three bowls and gives them out randomly. There are two red bowls and one yellow bowl. This information is shown on the tree diagram below.
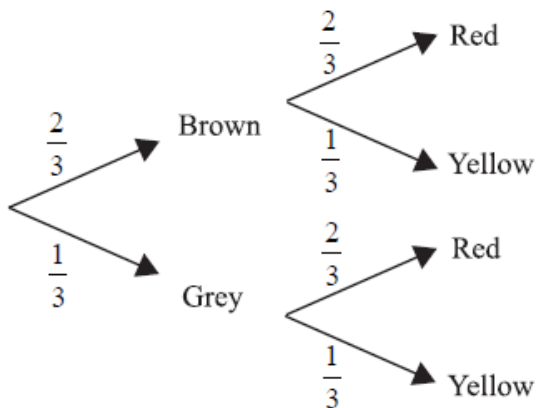
There are 49 mice in a pet shop.

30 mice are white.

27 mice are male.

18 mice have short tails.

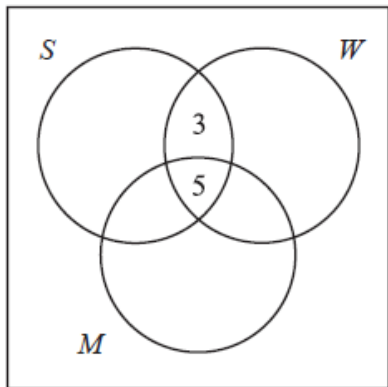8 mice are white and have short tails.

11 mice are male and have short tails.

7 mice are male but neither white nor short-tailed.

5 mice have all three characteristics and

2 have none.

Copy the diagram below to your examination script.

W represents white mice.
M represents male mice.
S represents short-tailed mice.

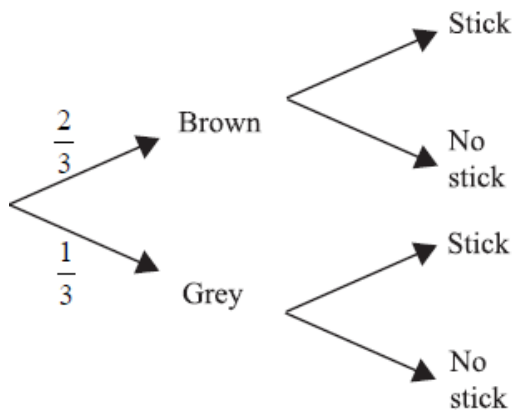i.a. One of the dogs is chosen at random. [3]

(i) Find P (the dog is grey and has the yellow bowl).

(ii) Find P (the dog does not get the yellow bowl).

i.b. Neil often takes the dogs to the park after they have eaten. He has noticed that the grey dog plays with a stick for a quarter of the time and both [9] brown dogs play with sticks for half of the time. This information is shown on the tree diagram below.

(i) Copy the tree diagram and add the four missing probability values on the branches that refer to playing with a stick.

During a trip to the park, one of the dogs is chosen at random.

(ii) Find P (the dog is grey or is playing with a stick, but not both).

(iii) Find P (the dog is grey given that the dog is playing with a stick).

(iv) Find P (the dog is grey and was fed from the yellow bowl and is not playing with a stick).

ii.a. Complete the diagram, using the information given in the question. [4]

ii.b. Find (i) $n(M \cap W)$ [3]

(ii) $n(M' \cup S)$

ii.c. Two mice are chosen without replacement. [2]

Find P (both mice are short-tailed).

On one day 180 flights arrived at a particular airport. The distance travelled and the arrival status for each incoming flight was recorded. The flight was then classified as on time, slightly delayed, or heavily delayed.

The results are shown in the following table.

		Distance travelled			
		At most 500 km	Between 500 km and 5000 km	At least 5000 km	TOTAL
Arrival Status	On time	19	17	16	52
	Slightly delayed	13	18	14	45
	Heavily delayed	28	15	40	83
	TOTAL	60	50	70	180

A χ^2 test is carried out at the 10 % significance level to determine whether the arrival status of incoming flights is independent of the distance travelled.

The critical value for this test is 7.779.

A flight is chosen at random from the 180 recorded flights.

a. State the alternative hypothesis. [1]

b. Calculate the expected frequency of flights travelling at most 500 km and arriving slightly delayed. [2]

c. Write down the number of degrees of freedom. [1]

d.i. Write down the χ^2 statistic. [2]

d.ii. Write down the associated p-value. [1]

e. State, with a reason, whether you would reject the null hypothesis. [2]

f. Write down the probability that this flight arrived on time. [2]

g. Given that this flight was not heavily delayed, find the probability that it travelled between 500 km and 5000 km. [2]

h. Two flights are chosen at random from those which were slightly delayed. [3]

Find the probability that each of these flights travelled at least 5000 km.

A survey of 400 people is carried out by a market research organization in two different cities, Buenos Aires and Montevideo. The people are asked which brand of cereal they prefer out of Chocos, Zucos or Fruti. The table below summarizes their responses.

	Chocos	Zucos	Fruti	Total
Buenos Aires	43	85	62	190
Montevideo	57	35	118	210
Total	100	120	180	400

The following table shows the cost in AUD of seven paperback books chosen at random, together with the number of pages in each book.

Book	1	2	3	4	5	6	7
Number of pages (x)	50	120	200	330	400	450	630
Cost (y AUD)	6.00	5.40	7.20	4.60	7.60	5.80	5.20

i.a. One person is chosen at random from those surveyed. Find the probability that this person   [4]

   (i) does not prefer Zucos;

   (ii) prefers Chocos, given that they live in Montevideo.

i.b. Two people are chosen at random from those surveyed. Find the probability that they both prefer Fruti.   [3]

i.c. The market research organization tests the survey data to determine whether the brand of cereal preferred is associated with a city. A chi-   [1]

   squared test at the 5% level of significance is performed.

   State the null hypothesis.

i.d. The market research organization tests the survey data to determine whether the brand of cereal preferred is associated with a city. A chi-   [1]

   squared test at the 5% level of significance is performed.

   State the number of degrees of freedom.

i.e. The market research organization tests the survey data to determine whether the brand of cereal preferred is associated with a city. A chi-   [2]

   squared test at the 5% level of significance is performed.

   Show that the expected frequency for the number of people who live in Montevideo and prefer Zucos is 63.

i.f. The market research organization tests the survey data to determine whether the brand of cereal preferred is associated with a city. A chi-   [2]

   squared test at the 5% level of significance is performed.

   Write down the chi-squared statistic for this data.

i.g. The market research organization tests the survey data to determine whether the brand of cereal preferred is associated with a city. A chi-   [2]

   squared test at the 5% level of significance is performed.

   State whether the market research organization would accept the null hypothesis. Clearly justify your answer.

ii.a. Plot these pairs of values on a scatter diagram. Use a scale of 1 cm to represent 50 pages on the horizontal axis and 1 cm to represent 1 AUD [3]

   on the vertical axis.

ii.b. Write down the linear correlation coefficient, r, for the data. [2]

ii.c. Stephen wishes to buy a paperback book which has 350 pages in it. He plans to draw a line of best fit to determine the price. State whether or not this is an appropriate method in this case and justify your answer. [2]

---

a. 180 people were interviewed and asked what types of transport they had used in the last year from a choice of airplane (A), train (T) or bus (B). The following information was obtained. [4]

47 had travelled by airplane

68 had travelled by train

122 had travelled by bus

25 had travelled by airplane and train

32 had travelled by airplane and bus

35 had travelled by train and bus

20 had travelled by all three types of transport

Draw a Venn diagram to show the above information.

b. Find the number of people who, in the last year, had travelled by [6]

(i)   bus only;

(ii)   both airplane and bus but not by train;

(iii) at least two types of transport;

(iv) none of the three types of transport.

c. A person is selected at random from those who were interviewed. [2]

Find the probability that the person had used only one type of transport in the last year.

d. Given that the person had used only one type of transport in the last year, find the probability that the person had travelled by airplane. [2]
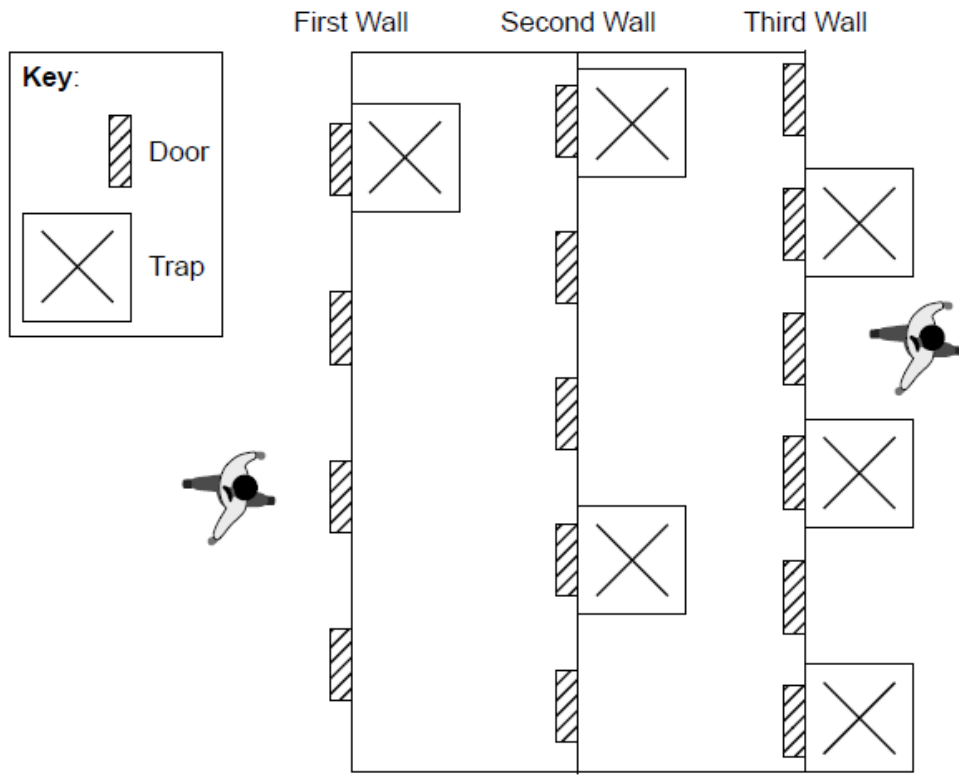

Contestants in a TV gameshow try to get through three walls by passing through doors without falling into a trap. Contestants choose doors at random.

If they avoid a trap they progress to the next wall.

If a contestant falls into a trap they exit the game before the next contestant plays.

Contestants are not allowed to watch each other attempt the game.

Key:

▨ Door

✕ Trap

The first wall has four doors with a trap behind one door.

Ayako is a contestant.

Natsuko is the second contestant.

The second wall has five doors with a trap behind two of the doors.

The third wall has six doors with a trap behind three of the doors.

The following diagram shows the branches of a probability tree diagram for a contestant in the game.

First Wall   Second Wall   Third Wall

Falls into trap

Falls into trap

Avoids trap

Falls into trap

Avoids trap
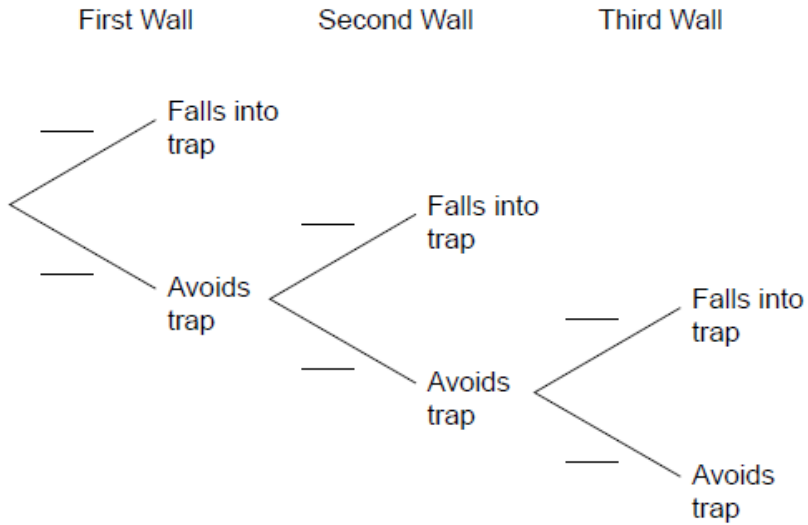
Avoids trap

a. Write down the probability that Ayako avoids the trap in this wall.   [1]

b. Find the probability that only one of Ayako and Natsuko falls into a trap while attempting to pass through a door **in the first wall**.   [3]

c. **Copy** the probability tree diagram and write down the relevant probabilities along the branches.   [3]

d.i. A contestant is chosen at random. Find the probability that this contestant fell into a trap while attempting to pass through a door in the second [2]

    wall.

d.ii A contestant is chosen at random. Find the probability that this contestant fell into a trap. [3]

e. 120 contestants attempted this game. [3]

    Find the expected number of contestants who fell into a trap while attempting to pass through a door in the third wall.

---

a. A group of students at Dune Canyon High School were surveyed. They were asked which of the following products: books (B), music (M) or [5]

    films (F), they downloaded from the internet.

    The following results were obtained:

    100 students downloaded music;
    95 students downloaded films;
    68 students downloaded films and music;
    52 students downloaded books and music;
    50 students downloaded films and books;
    40 students downloaded all three products;
    8 students downloaded books **only**;
    25 students downloaded none of the three products.

    Use the above information to complete a Venn diagram.

b. Calculate the number of students who were surveyed. [2]

c. i)   On your Venn diagram, shade the set $(F \cup M) \cap B'$. Do not shade any labels or values on the diagram. [3]

  ii)   Find $n\left((F \cup M) \cap B'\right)$.

d. A student who was surveyed is chosen at random. [6]

    Find the probability that

  (i)   the student downloaded music;

  (ii)   the student downloaded books, given that they had not downloaded films;

  (iii) the student downloaded at least two of the products.

e. Dune Canyon High School has 850 students. [2]

    Find the expected number of students at Dune Canyon High School that downloaded music.